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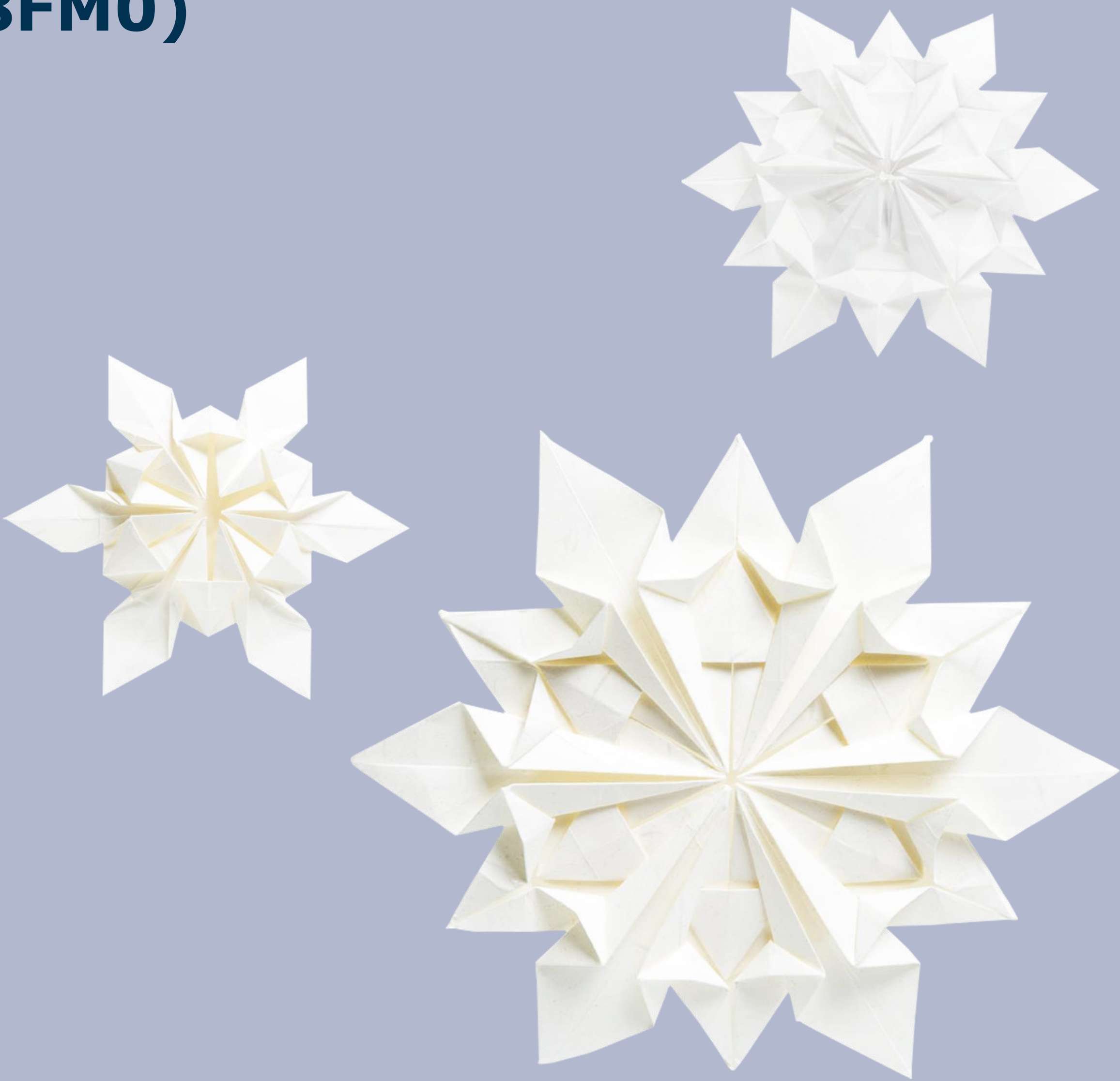
Pearson Edexcel Level 3 Advanced Subsidiary GCE in Further Mathematics (8FM0)



Sample Assessment Materials Model Answers – Decision Mathematics 1&2

First teaching from September 2017
First certification from June 2018

Pearson

Sample Assessment Materials Model Answers – Decision Mathematics 1&2

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Introduction

This booklet has been produced to support mathematics teachers delivering the new Pearson Edexcel Level 3 Advanced Subsidiary GCE in Mathematics (8FMO) specification for first teaching from September 2017.

This booklet looks at Sample Assessment Materials for AS Further Mathematics qualification, specifically at decision mathematics 1 and 2 questions, and is intended to offer model solutions with different methods explored.

Content of Decision Mathematics 1&2

Content	**AS level content**
	Decision Mathematics 1
Algorithms and graph theory	The general ideas of algorithms and the implementation of an algorithm given by flow chart or text. Bin packing, bubble sort and quick sort. Use of the order of the nodes to determine whether a graph is Eulerian, semi-Eulerian or neither.
Algorithms on graphs	The minimum spanning tree (minimum connector) problem. Prim's and Kruskal's algorithm. Dijkstra's algorithm for finding the shortest path.
Algorithms on graphs II	Algorithm for finding the shortest route around a network, travelling along every edge at least once and ending at the start vertex (The Route Inspection Algorithm).
Critical path analysis	Modelling of a project by an activity network, from a precedence table. Completion of the precedence table for a given activity network. Algorithm for finding the critical path. Earliest and latest event times. Earliest and latest start and finish times for activities. Identification of critical activities and critical path(s). Calculation of the total float of an activity. Construction of Gantt (cascade) charts. Each activity will require only one worker.
Linear programming	Formulation of problems as linear programs Graphical solution of two variable problems using objective line and vertex methods including cases where integer solutions are required.
	Decision Mathematics 2
Allocation (assignment) problems	Cost matrix reduction. Use of the Hungarian algorithm to find a least cost allocation. Modification of the Hungarian algorithm to deal with a maximum profit allocation.
Flows in networks	Cuts and their capacity. Use of the labelling procedure to augment a flow to determine the maximum flow in a network. Use of the max–flow min–cut theorem to prove that a flow is a maximum flow.
Game theory	Two person zero-sum games and the pay-off matrix. Identification of play safe strategies and stable solutions (saddle points). Optimal mixed strategies for a game with no stable solution by use of graphical methods for $2 \times n$ or $n \times 2$ games where $n = 1, 2, 3$ or 4.
Recurrence relations	Use of recurrence relations to model appropriate problems. Solution of first order linear homogeneous and nonhomogeneous recurrence relations.

Question 1

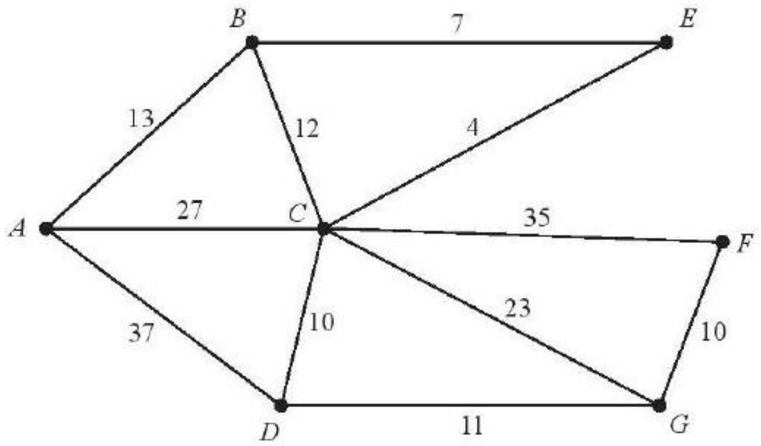


Figure 1

[*The total weight of the network is 189*]

Figure 1 represents a network of pipes in a building. The number on each arc is the length, in metres, of the corresponding pipe.

(a) Use Dijkstra's algorithm to find the shortest path from *A* to *F*. State the path and its length.
(5)

Using Dijkstra's algorithm:
Fill in the boxes on the answer book diagram as follows:
Since A is the start, give A final value 0. Write 1 in the order of labelling box for A to show that its final value was determined first.

A is connected to B, C and D.
Give B, C and D working values 13, 27, 37 respectively.
B has the lowest value, make this the final value. Write 2 in the order of labelling box for B to show that its final value was determined second.

B is connected to C and E.
Give C and E working values 13 + 12 = 25, 13 + 7 = 20. M1
E has the lower value, make this final. Write 3 in the order of labelling box for B to show that its final value was determined third.

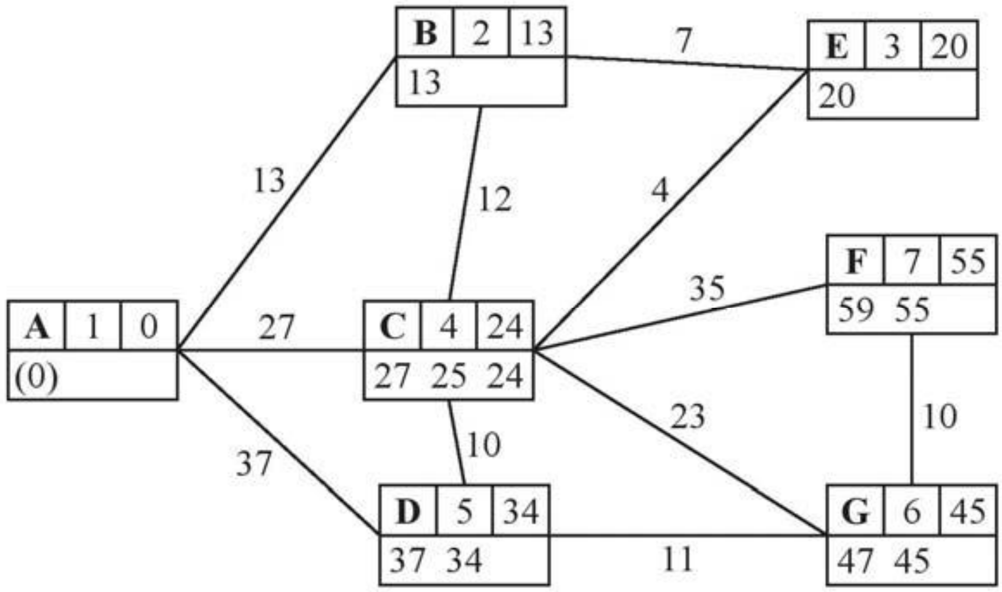
E is connected to C.
Give C final value 20 + 4 = 24. Write 4 in the order of labelling box for C.

C is connected to D, F and G.
Give D, F and G working values 24 + 10 = 34, 24 + 35 = 59, 24 + 23 = 47.
D has the lowest value, make the value of 34 final. Write 5 in the order of labelling box for D.

D is connected to G.
Give G final value 34 + 11 = 45. Write 6 in the order of labelling box for G.

G is connected to F.
Give F final value 45 +10 = 55. Write 7 in the order of labelling box for F.

The completed diagram is:



For all values correct (and in correct order) at A, B, C and D A1

For all values correct (and in correct order) at E, F & G A1

Key:

Vertex	Order of labelling	Final value
Working value		

Find the shortest path by tracing back from the final vertex linking vertices where the difference between the final labels is equal to the weight of the arc between the vertices.
The shortest path is: ABECDGF A1

Length of shortest path = 55 (metres) A1

On a particular day, Gabriel needs to check each pipe. A route of minimum length, which traverses each pipe at least once and which starts and finishes at *A*, needs to be found.

(b) Use an appropriate algorithm to find the pipes that will need to be traversed twice. You must make your method and working clear.

(4)

There are 4 odd nodes: A, B, D, G (all order 3)
The choices for repeated arcs are:
AB and DG or AD and BG or AG and BD — M1

AB: shortest route is direct = 13
DG: shortest route is direct = 11
AB + DG = 13 + 11 = 24

AD: shortest route is via BEC = 13+7+4+10 = 34
BG: shortest route is via ECD = 7+4+10+11 = 32
AD + BG = 34 + 32 = 66 — A1

AG: shortest route is via BECD = 13+7+4+10+11 = 45
BD: shortest route is via EC = 7+4+10 = 21
AG + BD = 45 + 21 = 66 — A1

So repeat arcs AB and DG as this is the shortest route. — A1

(c) State the minimum length of Gabriel's route.

(1)

Minimum length = 189 + 24 = 213 (metres) — B1

A new pipe, *BG*, is added to the network. A route of minimum length that traverses each pipe, including *BG*, needs to be found. The route must start and finish at *A*. Gabriel works out that the addition of the new pipe increases the length of the route by twice the length of *BG*.

(d) Calculate the length of *BG*. You must show your working.

(2)

Let BG = x
There are now only 2 odd nodes, A and D.
The shortest route between A and D is direct, so repeat arc AD = 34.

$189 + x + 34 = 213 + 2x$ — M1
$x = 10$
so BG = 10 (metres) — A1

Question 2

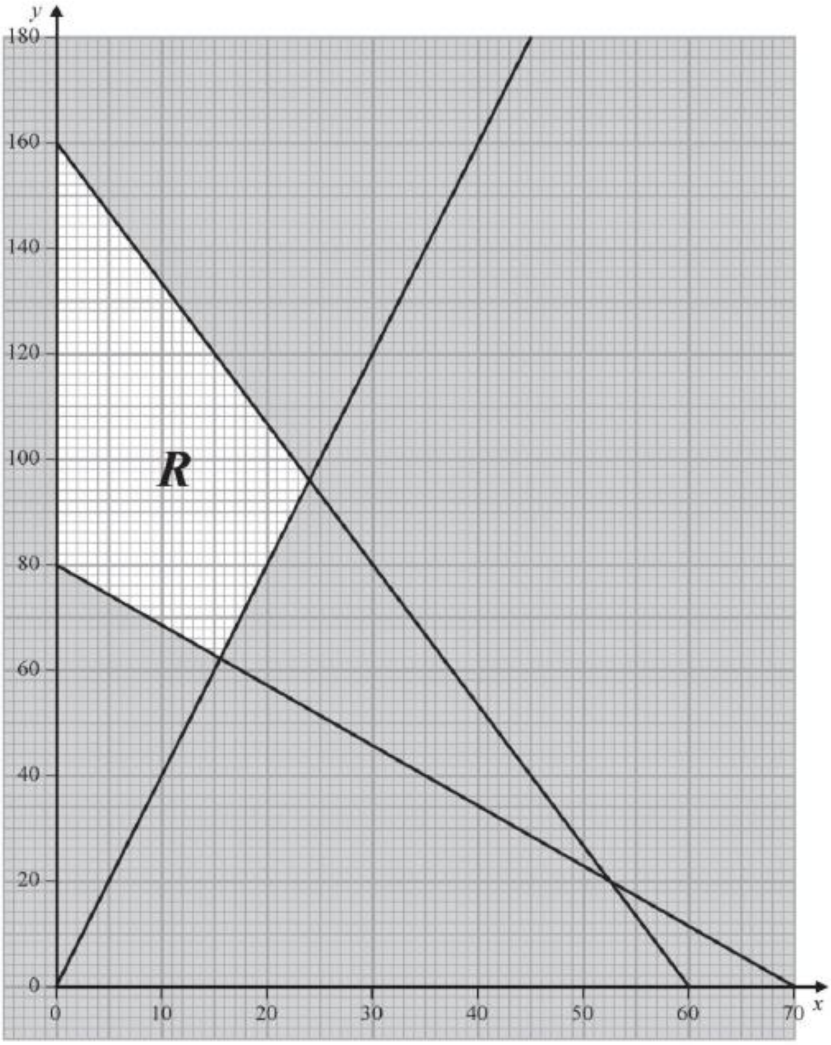


Figure 2

A teacher buys pens and pencils. The number of pens, x, and the number of pencils, y, that he buys can be represented by a linear programming problem as shown in Figure 2, which models the following constraints:

$$8x + 3y \leq 480$$

$$8x + 7y \geq 560$$

$$y \geq 4x$$

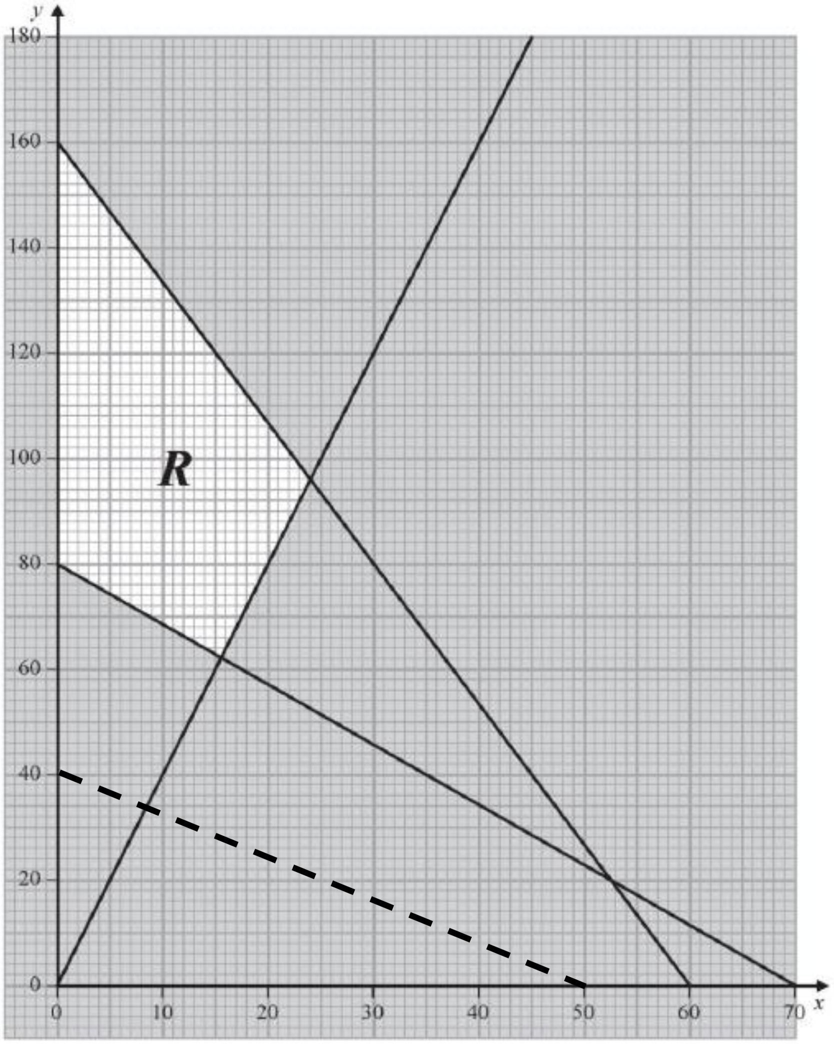
$$x, y \geq 0$$

The total cost, in pence, of buying the pens and pencils is given by $C = 12x + 15y$.

Determine the number of pens and the number of pencils which should be bought in order to minimise the total cost. You should make your method and working clear.

(7)

Either:
Drawing an objective line:
Picking any value for C e.g. let $C = 600$ then line is $12x + 15y = 600$
i.e. $y = -0.8x + 40$
Draw line on graph using points e.g. (0, 40) and (50, 0).
Objective line has been drawn as - - - - below.



Moving the objective line parallel to the original location and away from the origin to find the nearest point in the feasible region,
the first point of contact with the feasible region is at
the vertex which is approximately (15.5, 62)

Or:
Testing vertices:
The minimum cost must be at either (0, 80) or approximately (15.5, 62)
$C = 12 \times 0 + 15 \times 80 = 1200$
$C = 12 \times 15.5 + 15 \times 62 = 1116$ (minimum) M1
The other two vertices (0, 160) and (24, 96) will clearly give a larger C.

For both methods:

The point ≈ (15.5, 62) needs to be found exactly, so solve $y = 4x$ and $8x + 7y = 560$ simultaneously:

$8x + 7(4x) = 560$
$36x = 560$

$x = 15\frac{5}{9}$	M1
$y = 62\frac{2}{9}$	A1
Integer solutions are required for this problem, so test e.g. (15, 63), (16, 63), (15, 62) in $y \geq 4x$ and $8x + 7y \geq 560$	M1
$63 > 4 \times 15 = 60$, $8 \times 15 + 7 \times 63 = 561 > 560$ so (15, 63) is valid $63 < 4 \times 16 = 64$ so (16, 63) is not valid $8 \times 15 + 7 \times 62 = 554 < 560$ so (15, 62) is not valid	M1
$x = 15$, $y = 63$ is optimal	A1
So the teacher should buy 15 pens and 63 pencils	A1

Question 3

Activity	Time taken (days)	Immediately preceding activities
A	5	–
B	7	–
C	3	–
D	4	A, B
E	4	D
F	2	B
G	4	B
H	5	C, G
I	10	C, G

The table above shows the activities required for the completion of a building project.

For each activity, the table shows the time taken in days to complete the activity and the immediately preceding activities. Each activity requires one worker. The project is to be completed in the shortest possible time.

(a) Draw the activity network described in the table, using activity on arc. Your activity network must contain the minimum number of dummies only.

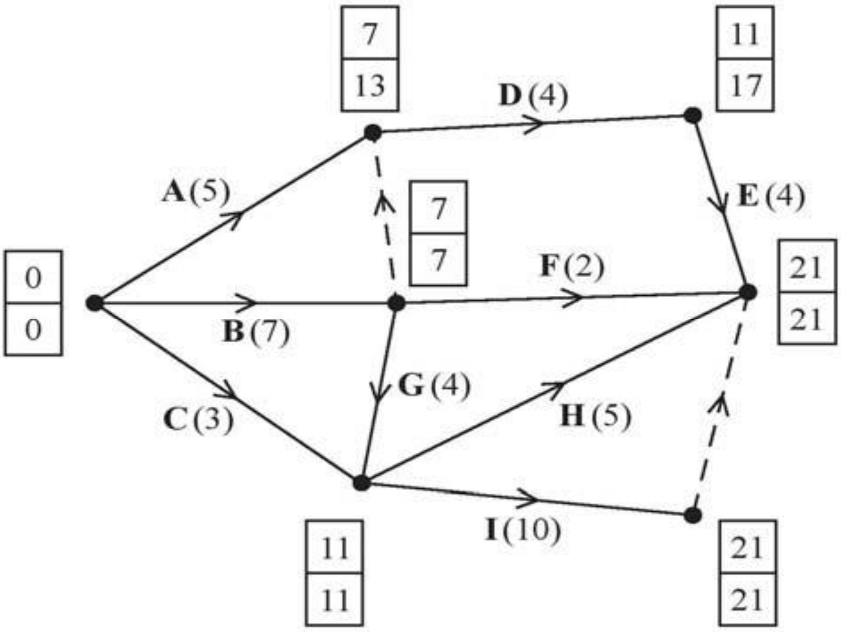
(3)

There must be one starting point, with arcs A, B and C.

The first dummy is needed because D depends on both A and B, and two activities cannot have both the same starting and finishing node.

The second dummy is needed because there must be only one finishing point.

At least 5 activities and one dummy, one start	M1
A,B,C,D,F,G and first dummy correct	A1
E,H,I correct, second dummy correct and one finish	A1



(b) (i) Show that the project can be completed in 21 days, showing your working.	
(ii) Identify the critical activities.	
	(4)

(i)
The boxes at each node on the diagram represent:

early event time
late event time

The early event time is the earliest time that all the activities preceding an event may be completed. The late event time is the latest time by which preceding events must be finished if the project is to be completed on time. M1
A1

The numbers at the end of activity E indicate this project can be completed in 21 days.
A1

(ii)
The critical activities are B, G and I. A1
(since they have the same numbers in both boxes i.e. early event time = late event time at these nodes)

Question 4

(a) Explain why it is not possible to draw a graph with exactly 5 nodes with orders 1, 3, 4, 4 and 5.

(1)

Either:
A graph cannot contain an odd number of odd nodes.

Or:
Number of arcs = (1+3+4+4+5) ÷ 2 = 8.5 which is not an integer. B1

A connected graph has exactly 5 nodes and contains 18 arcs. The orders of the 5 nodes are 2^{2x-1}, 2^x, $x + 1$, $2^{x+1} - 3$ and $11 - x$.

(b) (i) Calculate x.

(ii) State whether the graph is Eulerian, semi-Eulerian or neither. You must justify your answer.

(6)

(i)
(Each of the 18 arcs has two ends and therefore contribute 2 to the total of the degrees of the nodes, so)

$(2^{2x} - 1) + (2^x) + (x + 1) + (2^{x+1} - 3) + (11 - x) = 2 \times 18$ M1

$2^{2x} + 2^x + 2^x \times 2^1 + 8 = 36$

$2^{2x} + 3(2^x) - 28 = 0$

$(2^x - 4)(2^x + 7) = 0$

$2^x = 4$ or $2^x = -7$ (not possible)

M1

$x = 2$ A1

(ii)
The order of each node is: 15, 4, 3, 5, 9. M1

Therefore the graph is neither Eulerian, nor semi-Eulerian A1
as there are more than two odd nodes. A1

(c) Draw a graph which satisfies all of the following conditions:

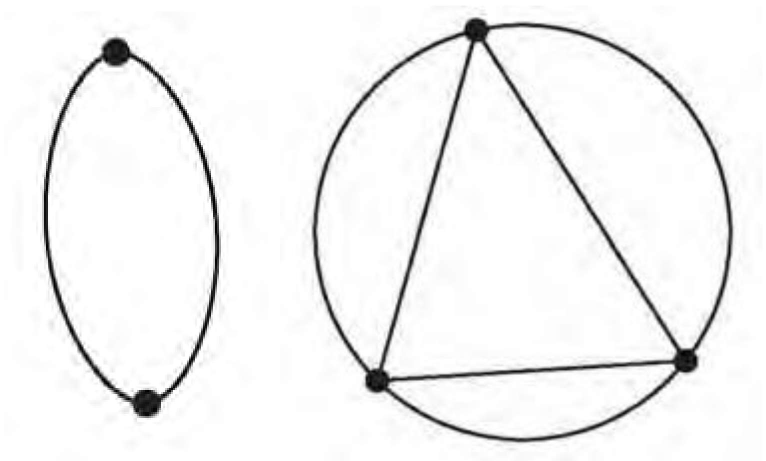
- The graph has exactly 5 nodes.
- The nodes have orders 2, 2, 4, 4 and 4.
- The graph is not Eulerian.

(2)

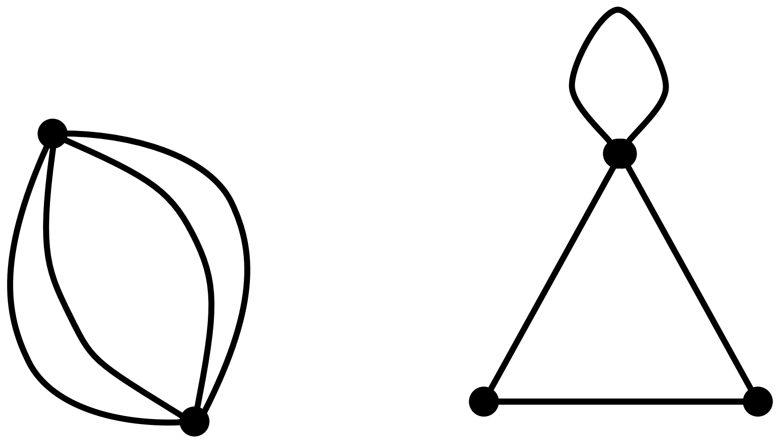
If all the nodes are even but the graph is not Eulerian, it must be disconnected.

M1

Therefore the graph is: A1



Alternative solution:



Question 5

Jonathan makes two types of information pack for an event, Standard and Value.

Each Standard pack contains 25 posters and 500 flyers.

Each Value pack contains 15 posters and 800 flyers.

He must use at least 150 000 flyers.

Between 35% and 65% of the packs must be Standard packs.

Posters cost 20p each and flyers cost 4p each.

Jonathan wishes to minimise his costs.

Let x and y represent the number of Standard packs and Value packs produced respectively.

Formulate this as a linear programming problem, stating the objective and listing the constraints as simplified inequalities with integer coefficients.

You should not attempt to solve the problem.

(5)

cost of standard pack (x) = $25 \times 0.2 + 500 \times 0.04 = 25$
cost of value pack (y) = $15 \times 0.2 + 800 \times 0.04 = 35$

minimise cost $C = 25x + 35y$ B1

subject to
at least 150000 flyers
$500x + 800y \geq 150000$
$5x + 8y \geq 1500$ B1

standard $\geq$ 35% of total
$x \geq \frac{7}{20}(x + y)$ M1
$20x \geq 7x + 7y$
$13x \geq 7y$

standard $\leq$ 65% of total
$x \leq \frac{13}{20}(x + y)$ M1
$20x \leq 13x + 13y$
$7x \leq 13y$

and $x \geq 0$, $y \geq 0$ A1

AS Decision Mathematics 2

Question 6

Six workers, *A*, *B*, *C*, *D*, *E* and *F*, are to be assigned to five tasks, *P*, *Q*, *R*, *S* and *T*.

Each worker can be assigned to at most one task and each task must be done by just one worker.

The time, in minutes, that each worker takes to complete each task is shown in the table below.

	P	*Q*	*R*	*S*	*T*
A	32	32	35	34	33
B	28	35	31	37	40
C	35	29	33	36	35
D	36	30	34	33	35
E	30	31	29	37	36
F	29	28	32	31	34

Reducing rows first, use the Hungarian algorithm to obtain an allocation which minimises the total time. You must explain your method and show the table after each stage.

(9)

As there are 6 workers and 5 tasks, there needs to be a dummy task X added, with all values = 40 (as largest value in table is BT = 40) to make the table square

	P	Q	R	S	T	X
A	32	32	35	34	33	40
B	28	35	31	37	40	40
C	35	29	33	36	35	40
D	36	30	34	33	35	40
E	30	31	29	37	36	40
F	29	28	32	31	34	40

B1

Reducing rows

	P	Q	R	S	T	X
A	0	0	3	2	1	8
B	0	7	3	9	12	12
C	6	0	4	7	6	11
D	6	0	4	3	5	10
E	1	2	0	8	7	11
F	1	0	4	3	6	12

Reducing columns M1

	P	Q	R	S	T	X
A	0	0	3	0	0	0
B	0	7	3	7	11	4
C	6	0	4	5	5	3
D	6	0	4	1	4	2
E	1	2	0	6	6	3
F	1	0	4	1	5	4

A1

All the 0's can be covered by just 4 horizontal or vertical lines.
No allocation can be made yet (as this is fewer than the number of rows and columns).
The smallest uncovered number is 1.
Subtract 1 (the smallest uncovered number) from all uncovered numbers.
Add 1 to the numbers at the intersection of two lines.

	P	Q	R	S	T	X
A	1	1	3	0	0	0
B	0	7	2	6	10	3
C	6	0	3	4	4	2
D	6	0	3	0	3	1
E	2	3	0	6	6	3
F	1	0	3	0	4	3

M1
A1

All the 0's can now be covered by 5 lines.
Still no allocation can be made.
The smallest uncovered number is again 1.
Subtract 1 from all uncovered numbers.
Add 1 to the numbers at the intersection of two lines.

	P	Q	R	S	T	X
A	2	2	3	1	0	0
B	0	7	1	6	9	2
C	6	0	2	4	3	1
D	6	0	2	0	2	0
E	3	4	0	7	6	3
F	1	0	2	0	3	2

M1
A1
A1

6 lines are now required to cover all the 0's.
Since there are 6 columns & rows also, this table is optimal.
So the allocation is:

A – T
B – P
C – Q
D – not allocated
E – R
F – S A1

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Question 7

In two-dimensional space, lines divide a plane into a number of different regions.

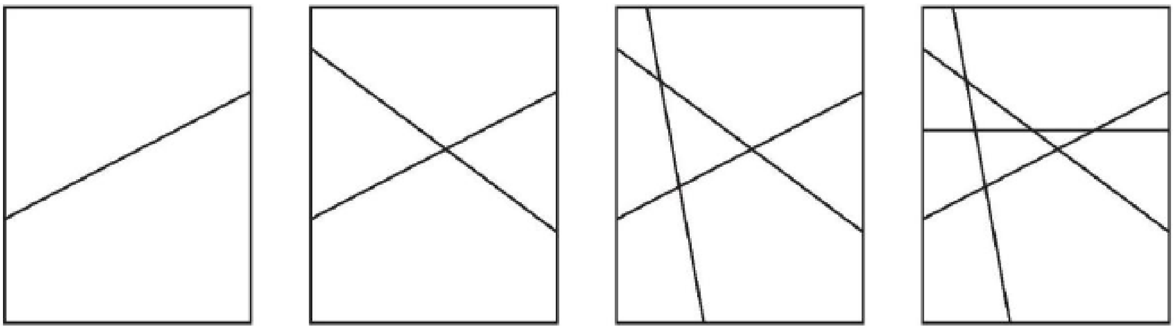


Figure 1 **Figure 2** **Figure 3** **Figure 4**

It is known that:

- One line divides a plane into 2 regions, as shown in Figure 1,
- Two lines divide a plane into a maximum of 4 regions, as shown in Figure 2,
- Three lines divide a plane into a maximum of 7 regions, as shown in Figure 3,
- Four lines divide a plane into a maximum of 11 regions, as shown in Figure 4.

(a) Complete the table in the answer book to show the maximum number of regions when five, six and seven lines divide a plane.

(1)

Number of Lines	1	2	3	4	5	6	7
Maximum Number of Regions	2	4	7	11	16	22	29

B1

(b) Find, in terms of u_n, the recurrence relation for u_{n+1}, the maximum number of regions when a plane is divided by $(n+1)$ lines, where $n \geq 1$.

(1)

$u_{n+1} = u_n + n + 1$ B1

(c) (i) Solve the recurrence relation for u_n.

(ii) Hence determine the maximum number of regions created when 200 lines divide a plane.
(3)

u_n	2	4	7	11	16	22	29
1st differences	2	3	4	5	6	7	
2nd differences	1	1	1	1	1		

As the 2nd differences are equal, u_n is a quadratic.
Let $u_n = an^2 + bn + c$

For 0 lines, there will be 1 region.
$n = 0$:
$1 = 0a + 0b + c$
$c = 1$

$n = 1$:
$2 = 1a + 1b + c$
$a + b = 1$ ---[1]

$n = 2$:
$4 = 4a + 2b + c$
$4a + 2b = 3$ ---[2]

Solve simultaneously, [2] – 2 × [1] gives
$2a = 1$ M1
$a = ½$
$b = ½$
$u_n = ½n^2 + ½n + 1$ A1
$u_n = ½n(n + 1) + 1$

$n = 200$:
$$u_{200} = \frac{1}{2}(200)(201) + 1$$
$= 20101$ A1

Question 8

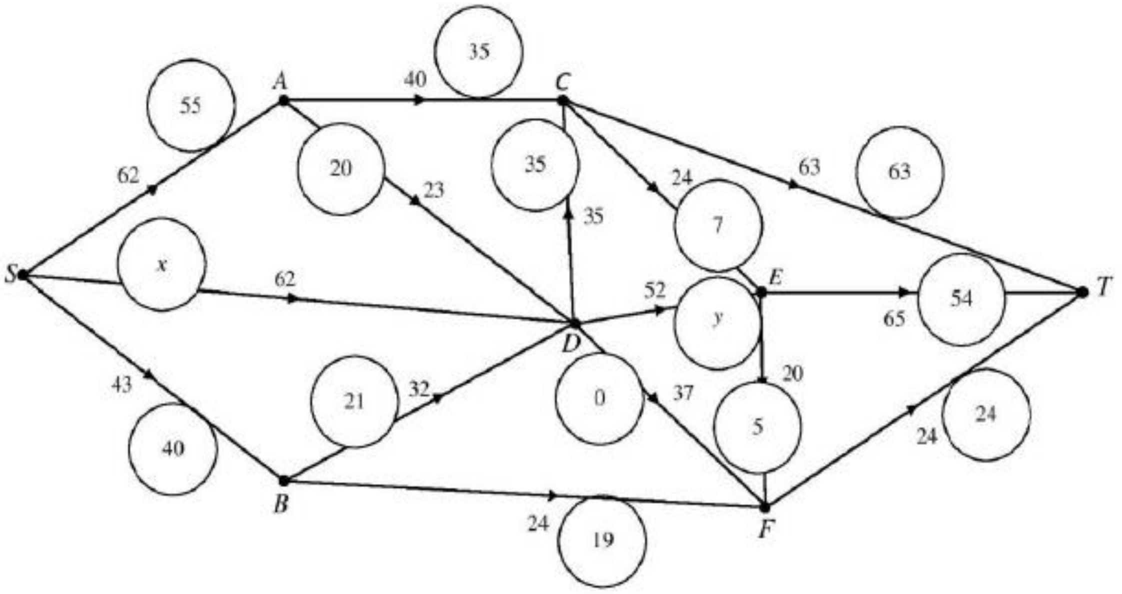


Figure 5

Figure 5 represents a network of corridors in a school. The number on each arc represents the maximum number of students, per minute, that may pass along each corridor at any one time.

At 11 a.m. on Friday morning, all students leave the hall (*S*) after assembly and travel to the cybercafe (*T*). The numbers in circles represent the initial flow of students recorded at 11 a.m. one Friday.

(a) State an assumption that has been made about the corridors in order for this situation to be modelled by a directed network.

(1)

The corridors must be one-way. B1

(b) Find the value of x and the value of y, explaining your reasoning.

(3)

flow out of S = flow into T
$55 + x + 40 = 63 + 54 + 24$ M1
$x = 46$ A1

flow into E = flow out of E
$7 + y = 54 + 5$
$y = 52$ A1

Five new students also attend the assembly in the hall the following Friday. They too need to travel to the cybercafe at 11 a.m. They wish to travel together so that they do not get lost. You may assume that the initial flow of students through the network is the same as that shown in Figure 5 above.

(c) (i) List all the flow augmenting routes from S to T that increase the flow by at least 5.

(ii) State which route the new students should take, giving a reason for your answer.

(3)

(i)
SACET can be increased by 5. M1
SDFET can be increased by 5. A1

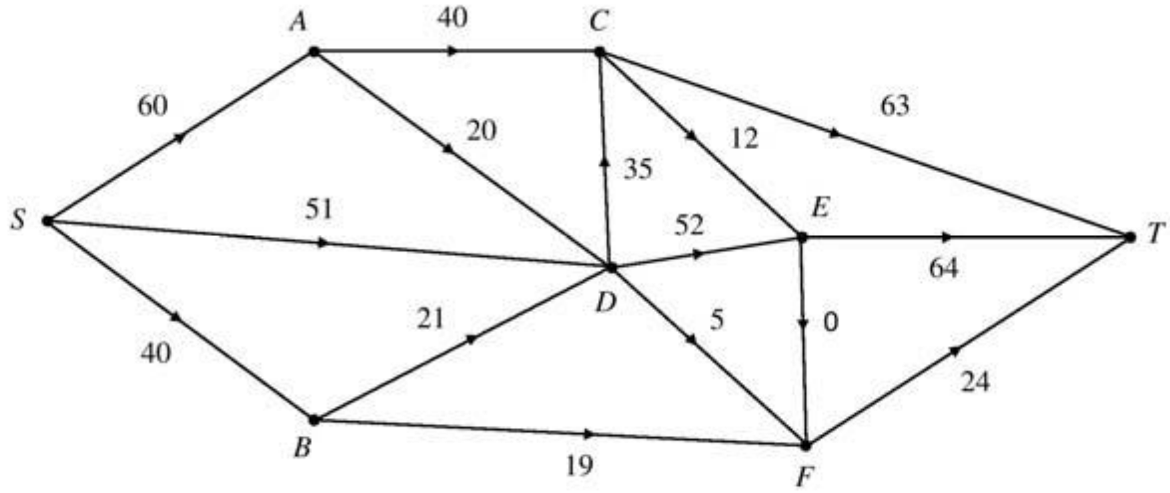
(ii)
Students must choose SACET, as they cannot travel
in the direction from F to E. A1

(d) Use the answer to part (*c*) to find a maximum flow pattern for this network.

(1)

Use both of the flow augmenting routes from part (c).
This increases the total capacity by 10 to give:

B1



(e) Prove that the answer to part (*d*) is optimal.

(3)

Using the max flow – min cut theorem:
A cut can be made through AC, DC, DE, EF, FT
Value of cut = 40 + 35 + 52 + 0 + 24 = 151 M1
Value of flow = 151 A1
These are equal, so it follows that the flow is optimal. A1

The school is intending to increase the number of students it takes but has been informed it cannot do so until it improves the flow of students at peak times. The school can widen corridors to increase their capacity, but can only afford to widen one corridor in the coming term.

(f) State, explaining your reasoning,

(i) which corridor they should widen,

(ii) the resulting increase of flow through the network.

(3)

Consider increasing the capacity of arcs in the minimum cut. B1
Increasing the capacity of any arc other than FT would not increase the flow by more than 1, as the total capacity into T is only 152.

Increasing the capacity on FT could increase the total flow by 16 (since the increased flow along SAD, SD and SBD could all be directed through DF to F). B1

Therefore the school should choose to widen FT, which could increase the flow through the network (corridors) by 16. B1

Question 9

A two person zero-sum game is represented by the following pay-off matrix for player A.

	B plays 1	B plays 2	B plays 3
A plays 1	4	1	2
A plays 2	2	4	3

(a) Verify that there is no stable solution.

(3)

	B plays 1	B plays 2	B plays 3	Row minima	
A plays 1	4	1	2	1	
A plays 2	2	4	3	2	$\leftarrow$ max = 2
Column maxima	4	4	3		
			$\uparrow$ min = 3		

M1
A1

Row maximin (2) $\neq$ Column minimax (3)
so there is no stable solution.
A1

(b) (i) Find the best strategy for player A.

(ii) Find the value of the game to her.

(9)

(i)
Let A play strategy 1 with probability p and strategy 2 with probability $1 - p$.
Then
if B plays strategy 1, M1
A's expected gains are $4p + 2(1 - p) = 2p + 2$ A1

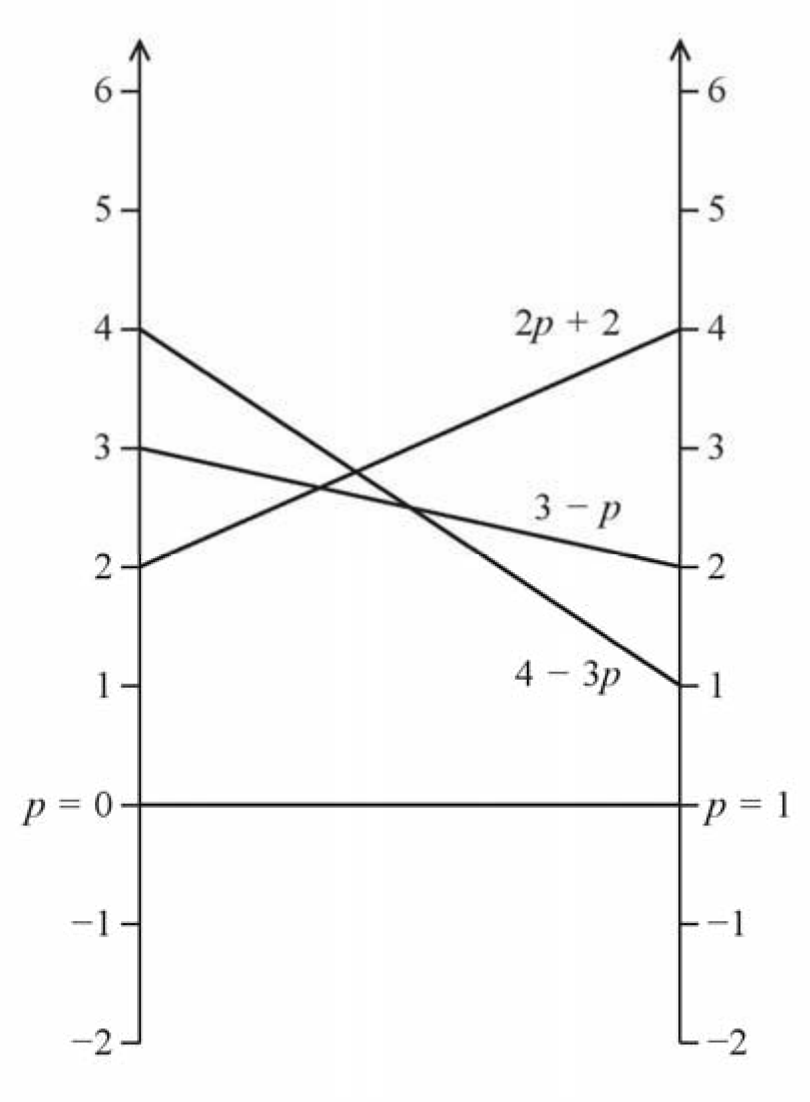
if B plays strategy 2,
A's expected gains are $1p + 4(1 - p) = 4 - 3p$

if B plays strategy 3,
A's expected gains are $2p + 3(1 - p) = 3 - p$ A1

Let the value of the game = v
Need to maximise v subject to

$v \leq 2p + 2 \qquad v \leq 4 - 3p \qquad v \leq 3 - p$

Draw the lines $v = 2p + 2$, $v = 4 - 3p$, $v = 3 - p$ on a probability expectation graph with $0 \leq p \leq 1$.



M1

A1

The maximum v occurs at the intersection of $v = 2p + 2$ and $v = 3 - p$.

$2p + 2 = 3 - p$

$3p = 1$

$p = \frac{1}{3}$ dM1

A1

Therefore player A should play strategy 1 for $\frac{1}{3}$ of the time and play strategy 2 for $\frac{2}{3}$ of the time.

A1

(ii)

The value of the game to player A is $v = 2p + 2 = 2(\frac{1}{3}) + 2 = 2\frac{2}{3}$ A1